

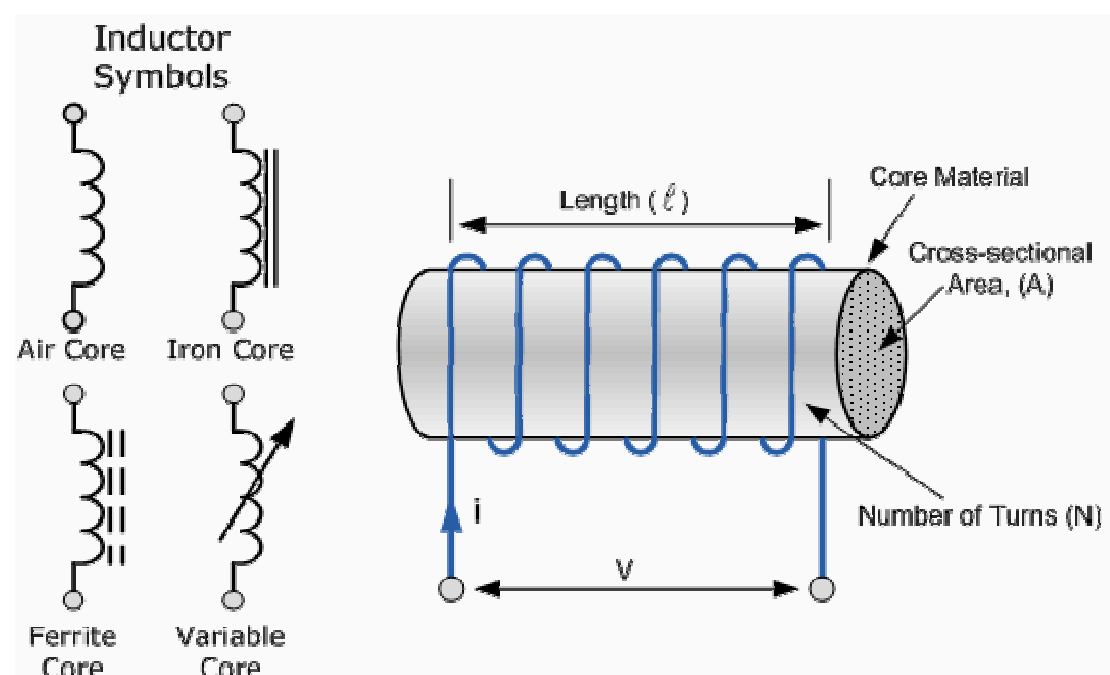
Electronics tutorial - Inductors

The Inductor

In its most basic form, an **Inductor** is simply a coil of wire. For most coils the current, (i) flowing through the coil produces a magnetic flux, ($N\Phi$) that is proportional to this flow of electrical current. In our tutorials about [Electromagnetism](#) we saw that when electrons flow through a conductor a magnetic flux is developed around the conductor producing a relationship between the direction of this flux around the conductor and the direction of the electrons flowing through the conductor, with this relationship being called, "Fleming's Left Hand Rule". But another important property of a wound coil exists, and that is a voltage is induced into the coil by this magnetic flux as it opposes or resists any changes in the electrical current flowing it.

The **Inductor**, also called a choke, is another passive type electrical component designed to take advantage of this relationship by producing a much stronger magnetic field than one that would be produced by a simple coil. Inductors are formed with wire tightly wrapped around a solid central core which can be either a straight cylindrical rod or a continuous loop or ring to concentrate their magnetic flux. The schematic symbol for an inductor is that of a coil of wire so therefore, a coil of wire can also be called an **Inductor**. Inductors are categorised according to the type of inner core with the different core types being distinguished by adding continuous or dotted parallel lines next to the wire coil as shown below.

Inductor Symbols



The current, i that flows through an inductor produces a magnetic flux that is proportional to it. But unlike a [Capacitor](#) which oppose a change of voltage across their plates, an inductor opposes the rate of change of current flowing through it due to the build up of self-induced energy within its magnetic field. In other words, inductors resist or oppose changes of current but will easily pass a steady state DC current. This ability of an inductor to resist changes in current and which also relates current, i with its magnetic flux linkage, $N\Phi$ as a constant of proportionality is called **Inductance** which is given the symbol **L** with units of **Henry, (H)** after Joseph Henry.

Because the Henry is a relatively large unit of inductance in its own right, for the smaller inductors sub-units of the Henry are used to denote its value. For example:

Prefix	Symbol	Multiplier	Power of Ten
milli	m	1/1,000	10^{-3}
micro	μ	1/1,000,000	10^{-6}
nano	n	1/1,000,000,000	10^{-9}

So to display the sub-units of the Henry we would use as an example:

- 1mH = 1 milli-Henry - which is equal to one thousandths (1/1000) of an Henry.
-
- 100uH = 100 micro-Henries - which is equal to 100 millionth's (1/1,000,000) of a Henry.

Inductors or coils are very common in electrical circuits and there are many factors which determine the inductance of a coil such as the shape of the coil, the number of turns of the insulated wire, the number of layers of wire, the spacing between the turns, the permeability of the core material, the size or cross-sectional area of the core etc, to name a few. An inductor coil has a central core area, (A) with a constant number of turns of wire per unit length, (l). So if a coil of N turns is linked by an amount of magnetic flux, Φ then the coil has a flux linkage of $N\Phi$ and any current, (i) that flows through the coil will produce an induced magnetic flux in the opposite direction to the flow of current. Then according to Faraday's Law, any change in this magnetic flux linkage produces a self-induced voltage in the single coil of:

$$V_L = N \frac{d\Phi}{dt} = \frac{\mu N^2 A}{l} \frac{di}{dt}$$

- Where:
- N is the number of turns
- A is the cross-sectional Area in m²
- Φ is the amount of flux in Webers

- μ is the Permeability of the core material
- l is the Length of the coil in meters
- di/dt is the Currents rate of change in amps/second

A time varying magnetic field induces a voltage that is proportional to the rate of change of the current producing it with a positive value indicating an increase in emf and a negative value indicating a decrease in emf. The equation relating this self-induced voltage, current and inductance can be found by substituting the $\mu N^2 A / l$ with L denoting the constant of proportionality called the **Inductance** of the coil.

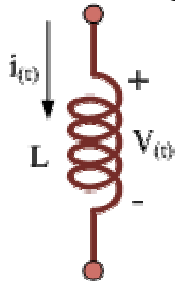
The relation between the flux in the inductor and the current flowing through the inductor is given as: $\Phi = Li$. As an inductor consists of a coil of conducting wire, this then reduces the above equation to give the self-induced emf, sometimes called the **back emf** induced in the coil too:

The back emf Generated by an Inductor

$$V_L(t) = \frac{d\phi}{dt} = \frac{dLi}{dt} = -L \frac{di}{dt}$$

- Where: L is the self-inductance and di/dt the rate of current change.

So from this equation we can say that the "self-induced emf = inductance x rate of current change" and a



Inductor Coil

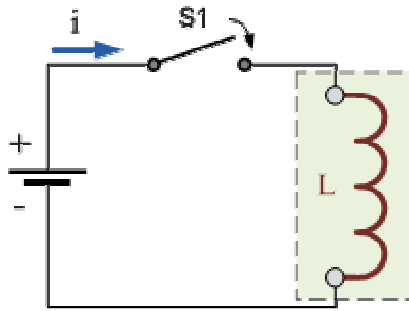
circuit has an inductance of one Henry will have an emf of one volt induced in the circuit when the current flowing through the circuit changes at a rate of one ampere per second.

One important point to note about the above equation. It only relates the emf produced across the inductor to changes in current because if the flow of inductor current is constant and not changing such as in a steady state DC current, then the induced emf voltage will be zero because the instantaneous rate of current change is zero, $di/dt = 0$.

With a steady state DC current flowing through the inductor and therefore zero induced voltage across it, the inductor acts as a short circuit equal to a piece of wire, in the presence of a continuous current. In other words, the opposition to the flow of current offered by an inductor is different between AC and DC circuits.

The Time Constant of an Inductor

We now know that the current can not change instantaneously in an inductor because for this to occur, the current would need to change by a finite amount in zero time which would result in the rate of current change being infinite, $di/dt = \infty$, making the induced emf infinite as well and infinite voltages do not exist. However, if the current flowing through an inductor changes very rapidly, such as with the operation of a switch, high voltages can be induced across the inductor's coil.



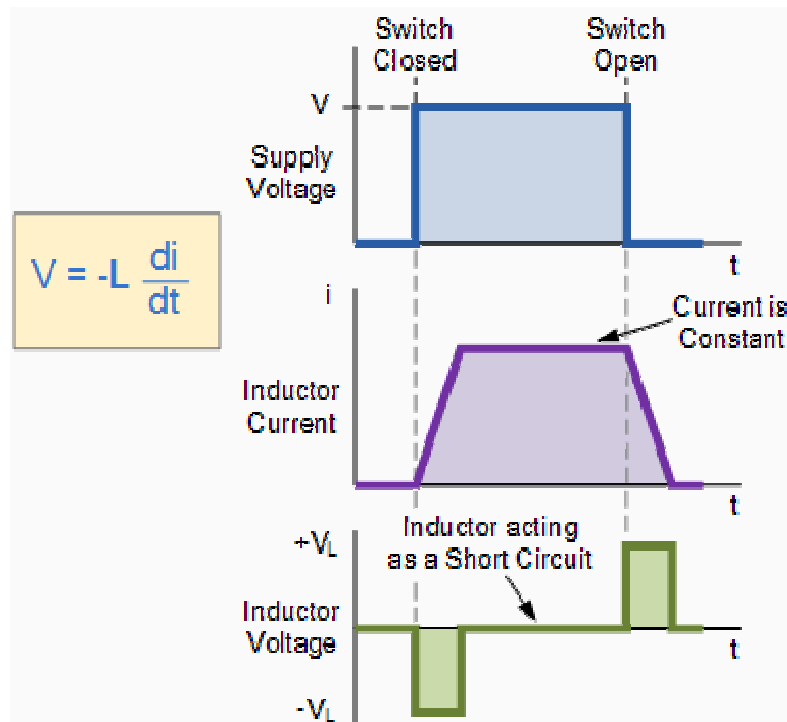
Consider the circuit of the inductor on the right. With the switch, (S1) open no current flows through the inductor coil, so the rate of change of current change (di/dt) in its coil is zero and therefore, zero self-induced emf exists across the inductor, ($V_L = 0$).

If we now close the switch ($t = 0$), a current will flow through the circuit and slowly rise to its maximum value at a rate determined by the inductance of the inductor. This rate of current flowing through the inductor multiplied by the inductor's inductance in Henry's, results in some fixed value self-induced emf being produced across the coil as determined by Faraday's equation above, $V_L = L di/dt$.

This self-induced emf across the inductor's coil, (V_L) fights against the applied voltage until the current reaches its maximum value and a steady state condition is reached. The current which now flows through the coil is determined only by the DC or "pure" resistance of the coil's windings as the reactance value of the coil has decreased to zero because the rate of change of current (di/dt) is zero in steady state. In other words, only the coil's DC resistance now exists to oppose the flow of current.

Likewise, if switch, (S1) is opened, the current flowing through the coil will start to fall but the inductor will again fight against this change and try to keep the current flowing at its previous value by inducing a voltage in the other direction. The slope of the fall will be negative and related to the inductance of the coil as shown below.

Current and Voltage in an Inductor



How much induced voltage will be produced by the inductor depends upon the rate of current change. In our tutorial about [Electromagnetic Induction](#), **Lenz's Law** stated that: "the direction of an induced emf is such that it will always opposes the change that is causing it". In other words, an induced emf will always **OPPOSE** the motion or change which started the induced emf in the first place. So with a decreasing current the voltage polarity will be acting as a source and with an increasing current the voltage polarity will be acting as a load. So for the same rate of current change through the coil, either increasing or decreasing the magnitude of the induced emf will be the same.

Example No1

A steady state direct current of 4 ampere passes through a solenoid coil of 0.5H. What would be the voltage induced in the coil if the switch opened for 10mS and the current flowing through the coil dropped to zero ampere.

$$V_L = L \frac{di}{dt} = 0.5 \frac{4}{0.01} = 200 \text{ volts}$$

Power and Energy in an Inductor

Power in an Inductor

We know that an inductor in a circuit opposes the flow of current, (i) through it because the flow of this current induces an emf that opposes it, Lenz's Law. Then

work has to be done by the external battery source in order to keep the current flowing against this induced emf. The instantaneous power used in forcing the current, (i) against this self-induced emf, (V_L) is given from above as:

$$V_{L(t)} = -L \frac{di}{dt}$$

Power in a circuit is given as, $P = V.I$ therefore:

$$P = v.i = \left(L \frac{di}{dt} \right) \times i = \frac{1}{2} L \frac{di^2}{dt} = \frac{d}{dt} \left[\frac{1}{2} L i^2 \right]$$

An ideal inductor has no resistance only inductance so $R = 0 \Omega$'s and therefore no power is dissipated within the coil, so we can say that an ideal inductor has zero power loss.

Energy in an Inductor

When power flows into an inductor, energy is stored in its magnetic field. When the current flowing through the inductor is increasing and di/dt becomes greater than zero, the instantaneous power in the circuit must also be greater than zero, ($P > 0$) ie, positive which means that energy is being stored in the inductor. Likewise, if the current through the inductor is decreasing and di/dt is less than zero then the instantaneous power must also be less than zero, ($P < 0$) ie negative which means that the inductor is returning energy back into the circuit. Then by integrating the equation for power above, the total magnetic energy which is always positive, being stored in the inductor is therefore given as:

Energy stored by an Inductor

$$W_{(t)} = \frac{1}{2} L i_{(t)}^2$$

- Where: W is in joules, L is in Henries and i is in Amperes

The energy is actually being stored within the magnetic field that surrounds the inductor by the current flowing through it. In an ideal inductor that has no resistance or capacitance, as the current increases energy flows into the inductor and is stored there within its magnetic field without loss, it is not released until the current decreases and the magnetic field collapses. Then in an alternating current, AC circuit an inductor is constantly storing and delivering energy on each and every cycle. If the current flowing through the inductor is constant as in a DC circuit, then there is no change in the stored energy as $P = Li(di/dt) = 0$.

So inductors can be defined as passive components as they can both store and deliver energy to the circuit, but they cannot generate energy. An ideal inductor is classed as lossless, meaning that it can store energy indefinitely as no energy is lost. However, real inductors will always have some resistance associated with the windings of the coil and whenever current flows through a resistance energy is lost in the form of heat due to [Ohm's Law](#), ($P = I^2 R$) regardless of whether the current is alternating or constant.

In the next tutorial about Inductors, we will see that the effective resistance of a coil is called [Inductance](#), and that inductance which as we now know is the characteristic of an electrical conductor that "opposes a change in the current", can either be internally induced, called self-inductance or externally induced, called mutual-inductance.

Inductance

Inductance is the name given to the property of a component that opposes the change of current flowing through it and even a straight piece of wire will have some inductance. Inductors do this by generating a self-induced emf within itself as a result of their changing magnetic field. When the emf is induced in the same circuit in which the current is changing this effect is called **Self-induction**, (L) but it is sometimes commonly called back-emf as its polarity is in the opposite direction to the applied voltage.

When the emf is induced into an adjacent component situated within the same magnetic field, the emf is said to be induced by **Mutual-induction**, (M) and mutual induction is the basic operating principle of transformers, motors, relays etc. Self inductance is a special case of mutual inductance, and because it is produced within a single isolated circuit we generally call self-inductance simply, **Inductance**. The basic unit of inductance is called the **Henry**, (H) after Joseph Henry, but it also has the units of **Webers per Ampere** ($1 H = 1 \text{ Wb/A}$).

Lenz's Law tells us that an induced emf generates a current in a direction which opposes the change in flux which caused the emf in the first place, the principle of action and reaction. Then we can accurately define **Inductance** as being *"a circuit will have an inductance value of one Henry when an emf of one volt is induced in the circuit were the current flowing through the circuit changes at a rate of one ampere/second"*. In other words, a coil has an inductance of one Henry when the current flowing through it changes at a rate of one ampere/second inducing a voltage of one volt in it and this definition can be presented as:

$$L = 1H = 1V \frac{dI}{dt} = 1 \frac{\text{volt}}{\text{ampere/second}}$$

Inductance, L is actually a measure of an inductor's "resistance" to the change of the current flowing through the circuit and the larger is its value in Henries, the lower will be the rate of current change.

We know from the previous tutorial about the [Inductor](#), that inductors are devices that can store their energy in the form of a magnetic field. Inductors are made from individual loops of wire combined to produce a coil and if the number of loops within the coil are increased, then for the same amount of current flowing through the coil, the magnetic flux will also increase. So by increasing the number of loops or turns within a coil, increases the coils inductance. Then the relationship between self-inductance, (L) and the number of turns, (N) and for a simple single layered coil can be given as:

Self Inductance of a Coil

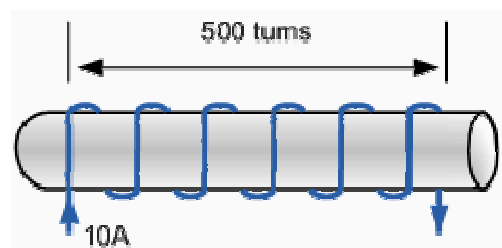
$$L = N \frac{\Phi}{I}$$

- Where:
- L is in Henries
- N is the Number of Turns
- Φ is the Magnetic Flux Linkage
- I is in Amperes

This expression can also be defined as the flux linkage divided by the current flowing through each turn. This equation only applies to linear magnetic materials.

Example No1

A hollow air cored inductor coil consists of 500 turns of copper wire which produces a magnetic flux of 10mWb when passing a DC current of 10 amps. Calculate the self-inductance of the coil in milli-Henries.



$$L = N \frac{\Phi}{I} = 500 \frac{0.01}{10} = 500\text{mH}$$

Example No2

Calculate the value of the self-induced emf produced in the same coil after a time of 10mS.

$$\text{emf} = L \frac{di}{dt} = 0.5 \frac{10}{0.01} = 500V$$

The self-inductance of a coil or to be more precise, the coefficient of self-inductance also depends upon the characteristics of its construction. For example, size, length, number of turns etc. It is therefore possible to have inductors with very high coefficients of self induction by using cores of a high permeability and a large number of coil turns. Then for a coil, the magnetic flux that is produced in its inner core is equal to:

$$\Phi = B.A$$

Where: Φ is the magnetic flux linkage, B is the flux density, and A is the area.

If the inner core of a long solenoid with N turns/metre is hollow, "air cored", the magnetic induction in its core will be given as.

$$B = \mu_0 H = \mu_0 \frac{N.I}{\ell}$$

Then by substituting these expressions in the first equation above for Inductance will give us:

$$L = N \frac{\Phi}{I} = N \frac{B.A}{I} = N \frac{\mu_0 . N . I}{\ell . I} . A$$

By cancelling out and grouping together like terms, then the final equation for the coefficient of self-inductance for an air cored coil (solenoid) is given as:

$$L = \mu_0 \frac{N^2 . A}{\ell}$$

- Where:
- L is in Henries
- μ_0 is the Permeability of Free Space ($4.\pi.10^{-7}$)
- N is the Number of turns
- A is the Inner Core Area ($\pi.r^2$) in m^2
- ℓ is the length of the Coil in metres

As the inductance of a coil is due to the magnetic flux around it, the stronger the magnetic flux for a given value of current the greater will be the inductance. So a coil of many turns will have a higher inductance value than one of only a few turns and therefore, the equation above will give inductance L as being proportional to the

number of turns squared N^2 . As well as increasing the number of coil turns, we can also increase inductance by increasing the coils diameter or making the core longer. In both cases more wire is required to construct the coil and therefore, more lines of force exists to produce the required back emf. The inductance of a coil can be increased further still if the coil is wound onto a ferromagnetic core, that is one made of a soft iron material, than one wound onto a non-ferromagnetic or hollow air core.

If the inner core is made of some ferromagnetic material such as soft iron, cobalt or nickel, the inductance of the coil would greatly increase because for the same amount of current flow the magnetic flux would be much stronger. This is because the lines of force would be more concentrated through the ferromagnetic core material as we saw in the [Electromagnets](#) tutorial. For example, if the core material has a relative permeability 1000 times greater than free space, $1000\mu_0$ such as soft iron or steel, than the inductance of the coil would be 1000 times greater so we can say that the inductance of a coil increases proportionally as the permeability of the core increases. Then for a coil wound around a former or core the inductance equation above would need to be modified to include the relative permeability μ_r of the new former material.

If the coil is wound onto a ferromagnetic core a greater inductance will result as the cores permeability will change with the flux density. However, depending upon the ferromagnetic material the inner cores magnetic flux may quickly reach saturation producing a non-linear inductance value and since the flux density around the coil depends upon the current flowing through it, inductance, L also becomes a function of current, i .

In the next tutorial about Inductors, we will see that the magnetic field generated by a coil can cause a current to flow in a second coil that is placed next to it. This effect is called [Mutual Inductance](#), and is the basic operating principal of transformers, motors and generators.

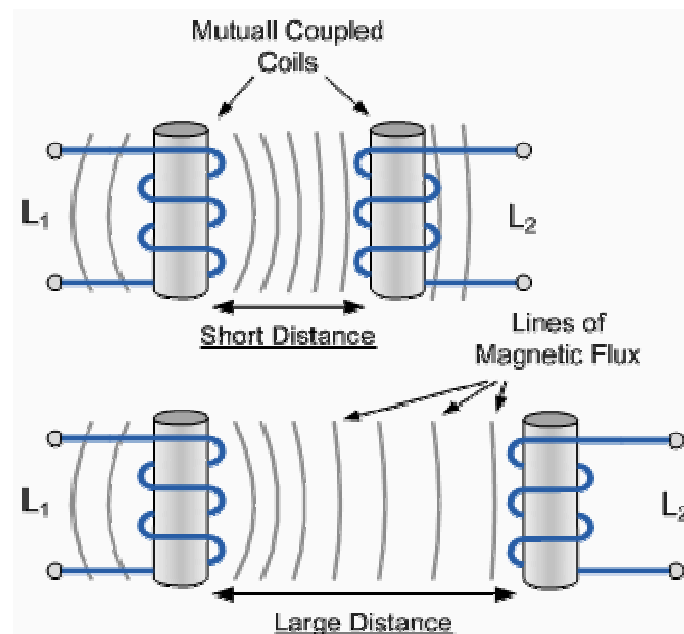
Mutual Inductance of Two Coils

In the previous tutorial we saw that an inductor generates an induced emf within itself as a result of the changing magnetic field around its own turns, and when this emf is induced in the same circuit in which the current is changing this effect is called **Self-induction**, (L). However, when the emf is induced into an adjacent coil situated within the same magnetic field, the emf is said to be induced magnetically, inductively or by **Mutual induction**, symbol (M). Then when two or more coils are magnetically linked together by a common magnetic flux they are said to have the property of **Mutual Inductance**.

Mutual Inductance is the basic operating principal of transformers, motors, generators and any other electrical component that interacts with another magnetic field. Then we can define mutual induction as the current flowing in one coil induces an emf in an adjacent coil. But mutual inductance can also be a bad thing as "stray" or "leakage" inductance from a coil can interfere with the operation of another adjacent component by means of electromagnetic induction, so some form of electrical screening to a ground potential may be required.

The amount of mutual inductance that links one coil to another depends very much on the relative positioning of the two coils. If one coil is positioned next to the other coil so that their physical distance apart is small, then nearly nearly all of the magnetic flux generated by the first coil will interact with the coil turns of the second coil inducing a relatively large emf and therefore producing a large mutual inductance value. Likewise, if the two coils are farther apart from each other or at different angles, the amount of induced magnetic flux from the first coil into the second will be weaker producing a much smaller induced emf and therefore a much smaller mutual inductance value. So the effect of mutual inductance is very much dependant upon the relative positions or spacing, (S) of the two coils and this is demonstrated below.

Mutual Inductance between Coils

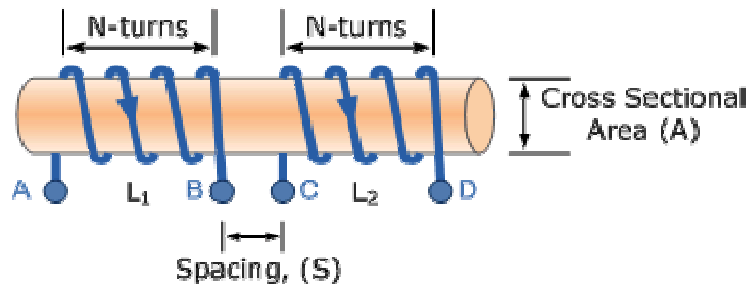


The mutual inductance that exists between the two coils can be greatly increased by positioning them on a common soft iron core or by increasing the number of turns of either coil as would be found in a transformer. If the two coils are tightly wound one on top of the other over a common soft iron core unity coupling is said to exist between them as any losses due to the leakage of flux will be extremely small. Then assuming a perfect flux linkage between the two coils the mutual inductance that exists between them can be given as.

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{\ell}$$

- Where:
- μ_0 is the permeability of free space ($4\pi \cdot 10^{-7}$)
- μ_r is the relative permeability of the soft iron core
- N is in the number of coil turns
- A is in the cross-sectional area in m^2
- ℓ is the coils length in meters

Mutual Induction



Here the current flowing in coil one, L₁ sets up a magnetic field around itself with some of these magnetic field lines passing through coil two, L₂ giving us mutual inductance. Coil one has a current of I₁ and N₁ turns while, coil two has N₂ turns. Therefore, the mutual inductance, M₁₂ of coil two that exists with respect to coil one depends on their position with respect to each other and is given as:

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$

Likewise, the flux linking coil one, L₁ when a current flows around coil two, L₂ is exactly the same as the flux linking coil two when the same current flows around coil one above, then the mutual inductance of coil one with respect of coil two is defined as M₂₁. This mutual inductance is true irrespective of the size, number of turns, relative position or orientation of the two coils. Because of this, we can write the mutual inductance between the two coils as: M₁₂ = M₂₁ = M.

We remember from our tutorials on [Electromagnets](#) that the self inductance of each individual coil is given as:

$$L_1 = \frac{\mu_0 \mu_r N_1^2 A}{\ell} \quad \text{and} \quad L_2 = \frac{\mu_0 \mu_r N_2^2 A}{\ell}$$

Then by cross-multiplying the two equations above, the mutual inductance that exists between the two coils can be expressed in terms of the self inductance of each coil.

$$M^2 = L_1 L_2$$

giving us a final and more common expression for the mutual inductance between two coils as:

Mutual Inductance Between Coils

$$M = \sqrt{L_1 L_2} \quad H$$

However, the above equation assumes zero flux leakage and 100% magnetic coupling between the two coils, L_1 and L_2 . In reality there will always be some loss due to leakage and position, so the magnetic coupling between the two coils can never reach or exceed 100%, but can become very close to this value in some special inductive coils. If some of the total magnetic flux links with the two coils, this amount of flux linkage can be defined as a fraction of the total possible flux linkage between the coils. This fractional value is called the **coefficient of coupling** and is given the letter k .

Coupling Coefficient

Generally, the amount of inductive coupling that exists between the two coils is expressed as a fractional number between 0 and 1 instead of a percentage (%) value, where 0 indicates zero or no inductive coupling, and 1 indicating full or maximum inductive coupling. In other words, if $k = 1$ the two coils are perfectly coupled, if $k > 0.5$ the two coils are said to be tightly coupled and if $k < 0.5$ the two coils are said to be loosely coupled. Then the equation above which assumes a perfect coupling can be modified to take into account this coefficient of coupling, k and is given as:

Coupling Factor Between Coils

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad \text{or} \quad M = k \times \sqrt{L_1 L_2} \quad H$$

When the coefficient of coupling, k is equal to 1, (unity) such that all the lines of flux of one coil cuts all of the turns of the other, the mutual inductance is equal to the geometric mean of the two individual inductances of the coils. So when the two inductances are equal and L_1 is equal to L_2 , the mutual inductance that exists between the two coils can be defined as:

$$M = \sqrt{L_1 L_2} = L$$

Example No1

Two inductors whose self-inductances are given as 75mH and 55mH respectively, are positioned next to each other on a common magnetic core so that 75% of the lines of flux from the first coil are cutting the second coil. Calculate the total mutual inductance that exists between them.

$$M = k\sqrt{L_1 L_2}$$

$$M = 0.75\sqrt{75\text{mH} \times 55\text{mH}} = 48.2\text{mH}$$

In the next tutorial about Inductors, we look at connecting together [Inductors in Series](#) and the affect this combination has on the circuits mutual inductance, total inductance and their induced voltage

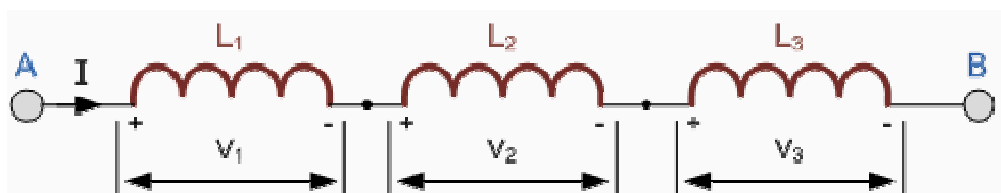
Connecting Inductors in Series

Inductors can be connected together in either a series connection, a parallel connection or combinations of both series and parallel together, to produce more complex networks whose overall inductance is a combination of the individual inductors. However, there are certain rules for connecting inductors in series or parallel and these are based on the fact that no mutual inductance or magnetic coupling exists between the individual inductors.

Inductors in Series

Inductors are said to be connected in "Series" when they are daisy chained together in a straight line, end to end. In the [Resistors in Series](#) tutorial we saw that the different values of the resistances just "add" together and this is also true of inductance. Inductors in series are added together because the number of coil turns is effectively increased, with the total circuit inductance L_T being equal to the sum of all the individual inductances added together.

Inductor in Series Circuit



The current, (I) that flows through the first inductor, L_1 has no other way to go but pass through the second inductor and the third and so on. Then, inductors in series have a **Common Current** flowing through them, for example:

$$I_{L1} = I_{L2} = I_{L3} = I_{AB} \dots \text{etc.}$$

In the example above, the inductors L_1 , L_2 and L_3 are all connected together in series between points A and B. The sum of the individual voltage drops across each inductor can be found using Kirchoff's Voltage Law (KVL) where, $V_T = V_1 + V_2 + V_3$ and we

know from the previous tutorials on inductance that the self-induced emf across an inductor is given as: $V = L \frac{di}{dt}$.

So by taking the values of the individual voltage drops across each inductor in our example above, the total inductance for the series combination is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when connecting inductors in series and this is given as:

Inductors in Series Equation

$$L_{\text{total}} = L_1 + L_2 + L_3 + \dots + L_n \text{ etc.}$$

Then the total inductance of the series chain can be found by simply adding together the individual inductances of the inductors in series just like adding together resistors in series. However, the above equation only holds true when there is "NO" mutual inductance or magnetic coupling between two or more of the inductors, (they are magnetically isolated from each other).

One important point to remember about inductors in series circuits, the total inductance (L_T) of any two or more inductors connected together in series will always be **GREATER** than the value of the largest inductor in the series chain.

Example No 1

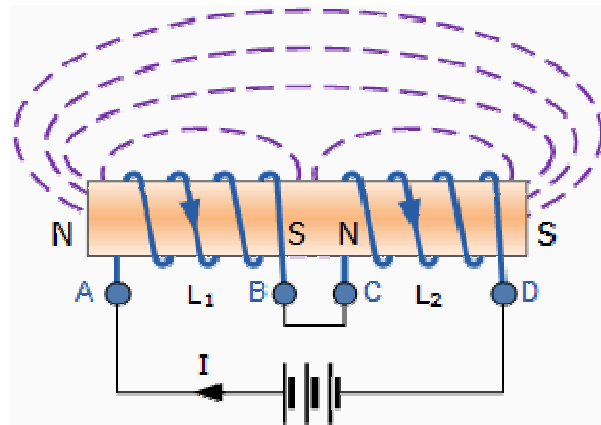
Three inductors of 10mH, 40mH and 50mH are connected together in a series combination with no mutual inductance between them. Calculate the total inductance of the series combination.

$$L_T = L_1 + L_2 + L_3 = 10\text{mH} + 40\text{mH} + 50\text{mH} = 100\text{mH}$$

Mutually Connected Inductors in Series

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other. Mutually connected inductors in series can be classed as either "Aiding" or "Opposing" the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be **Cumulatively Coupled**. If the current flows through the coils in opposite directions then the coils are said to be **Differentially Coupled** as shown below.

Cumulatively Coupled Series Inductors



While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil, L_1 and L_2 respectively will be the same as before but with the addition of M denoting the mutual inductance.

Then the total emf induced into the cumulatively coupled coils is given as:

$$L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2 \left(M \frac{di}{dt} \right)$$

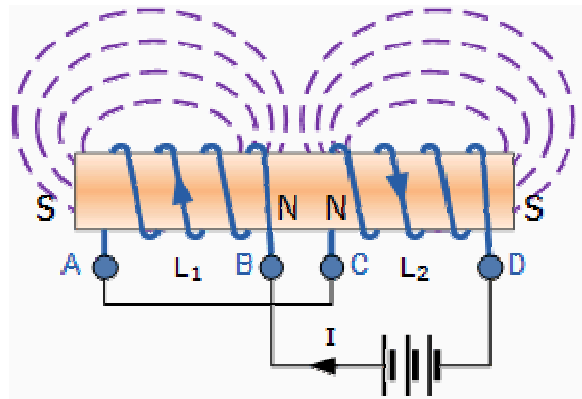
- Where: $2M$ represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1 .

By dividing through the above equation by di/dt we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

$$L_{\text{total}} = L_1 + L_2 + 2M$$

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil as shown below.

Differentially Coupled Series Inductors



The emf that is induced into coil 1 by the effect of the mutual inductance of coil 2 is in opposition to the self-induced emf in coil 1 as now the same current passes through each coil in opposite directions. To take account of this cancelling effect a minus sign is used with M when the magnetic field of the two coils are differentially connected giving us the final equation for calculating the total inductance of a circuit when the inductors are differentially connected as:

$$L_{\text{total}} = L_1 + L_2 - 2M$$

Then the final equation for inductively coupled inductors in series is given as:

$$L_T = L_1 + L_2 \pm 2M$$

Example No2

Two inductors of 10mH respectively are connected together in a series combination so that their magnetic fields aid each other giving cumulative coupling. Their mutual inductance is given as 5mH. Calculate the total inductance of the series combination.

$$L_T = L_1 + L_2 + 2M$$

$$L_T = 10\text{mH} + 10\text{mH} + 2(5\text{mH})$$

$$L_T = 30\text{mH}$$

Example No3

Two coils connected in series have a self-inductance of 20mH and 60mH respectively. The total inductance of the combination was found to be 100mH. Determine the amount of mutual inductance that exists between the two coils assuming that they are aiding each other.

$$L_T = L_1 + L_2 \pm 2M$$

$$100 = 20 + 60 + 2M$$

$$2M = 100 - 20 - 60$$

$$\therefore M = \frac{20}{2} = 10\text{mH}$$

Inductors in Series Summary

We now know that we can connect together **inductors in series** to produce a total inductance value, L_T equal to the sum of the individual values, they add together, similar to connecting together resistors in series. However, mutually connected inductors in series are classed as either "aiding" or "opposing" this total inductance depending whether the coils are cumulatively coupled (in the same direction) or differentially coupled (in opposite direction). In the next tutorial about Inductors, we will see that the position of the coils when connecting together [Inductors in Parallel](#) also affects the total inductance, L_T of the circuit.

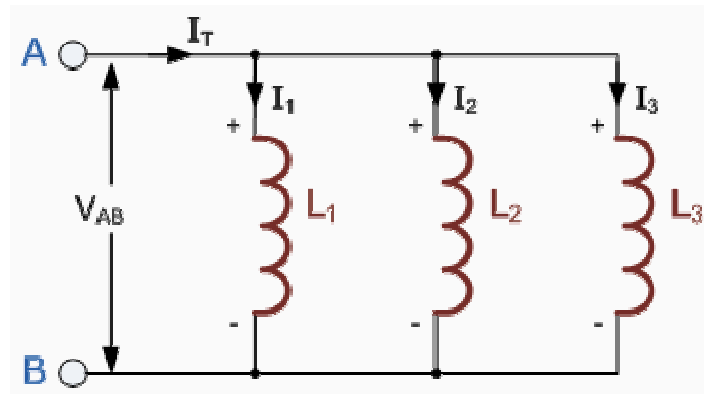
Inductors in Parallel

Inductors are said to be connected together in "**Parallel**" when both of their terminals are respectively connected to each terminal of the other inductor or inductors. The voltage drop across all of the inductors in parallel will be the same. Then, **Inductors in Parallel** have a **Common Voltage** across them and in our example below the voltage across the inductors is given as:

$$V_{L1} = V_{L2} = V_{L3} = V_{AB} \dots \text{etc}$$

In the following circuit the inductors L_1 , L_2 and L_3 are all connected together in parallel between the two points A and B.

Inductors in Parallel Circuit



In the previous series inductors tutorial, we saw that the total inductance, L_T of the circuit was equal to the sum of all the individual inductors added together. For inductors in parallel the equivalent circuit inductance L_T is calculated differently.

The sum of the individual currents flowing through each inductor can be found using Kirchhoff's Current Law (KCL) where, $I_T = I_1 + I_2 + I_3$ and we know from the previous tutorials on inductance that the self-induced emf across an inductor is given as:

$$V = L \, di/dt$$

Then by taking the values of the individual currents flowing through each inductor in our circuit above, and substituting the current i for $i_1 + i_2 + i_3$ the voltage across the parallel combination is given as:

$$V_{AB} = L_T \frac{d}{dt}(i_1 + i_2 + i_3) = L_T \left(\frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} \right)$$

By substituting di/dt in the above equation with v/L gives:

$$V_{AB} = L_T \left(\frac{v}{L_1} + \frac{v}{L_2} + \frac{v}{L_3} \right)$$

We can reduce it to give a final expression for calculating the total inductance of a circuit when connecting inductors in parallel and this is given as:

Parallel Inductor Equation

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots + \frac{1}{L_N}$$

Here, like the calculations for parallel resistors, the reciprocal ($1/L_n$) value of the individual inductances are all added together instead of the inductances themselves. But again as with series connected inductances, the above equation only holds true when there is "NO" mutual inductance or magnetic coupling between two or more of

the inductors, (they are magnetically isolated from each other). Where there is coupling between coils, the total inductance is also affected by the amount of coupling.

This method of calculation can be used for calculating any number of individual inductances connected together within a single parallel network. If however, there are only two individual inductors in parallel then a much simpler and quicker formula can be used to find the total inductance value, and this is:

$$L_T = \frac{L_1 \times L_2}{L_1 + L_2}$$

One important point to remember about inductors in parallel circuits, the total inductance (L_T) of any two or more inductors connected together in series will always be **LESS** than the value of the smallest inductance in the parallel chain.

Example No1

Three inductors of 60mH, 120mH and 75mH are connected together in a parallel combination with no mutual inductance between them. Calculate the total inductance of the parallel combination.

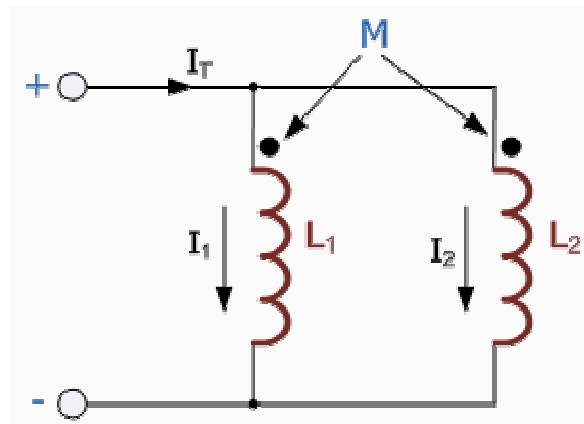
$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{60\text{mH}} + \frac{1}{120\text{mH}} + \frac{1}{75\text{mH}} = 26\text{mH}$$

Mutually Coupled Inductors in Parallel

When inductors are connected together in parallel so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling that exists between the coils. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other. Mutually connected inductors in parallel can be classed as either "aiding" or "opposing" the total inductance with parallel aiding connected coils increasing the total equivalent inductance and parallel opposing coils decreasing the total equivalent inductance compared to coils that have zero mutual inductance.

Mutual coupled parallel coils can be shown as either connected in an aiding or opposing configuration by the use of polarity dots or polarity markers as shown below.

Parallel Aiding Inductors



The voltage across the two parallel aiding inductors above must be equal since they are in parallel so the two currents, i_1 and i_2 must vary so that the voltage across them stays the same. Then the total inductance, L_T for two parallel aiding inductors is given as:

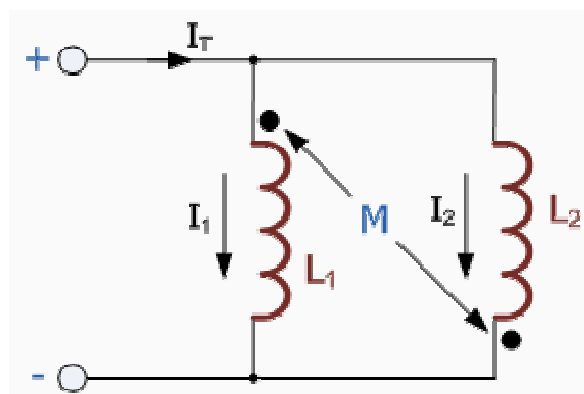
$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

- Where: $2M$ represents the influence of coil L_1 on L_2 and likewise coil L_2 on L_1 .

If the two inductances are equal and the magnetic coupling is perfect such as in a toroidal circuit, then the equivalent inductance of the two inductors in parallel is L as $L_T = L_1 = L_2 = M$. However, if the mutual inductance between them is zero, the equivalent inductance would be $L \div 2$ the same as for two self-induced inductors in parallel.

If one of the two coils was reversed with respect to the other, we would then have two parallel opposing inductors and the mutual inductance, M that exists between the two coils will have a cancelling effect on each coil instead of an aiding effect as shown below.

Parallel Opposing Inductors



Then the total inductance, L_T for two parallel opposing inductors is given as:

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

This time, if the two inductances are equal in value and the magnetic coupling is perfect between them, the equivalent inductance and also the self-induced emf across the inductors will be zero as the two inductors cancel each other out. This is because as the two currents, i_1 and i_2 flow through each inductor in turn the total mutual flux generated between them is zero because these two flux's produced by each inductor are both equal in magnitude but in opposite directions. Then the two coils effectively become a short circuit to the flow of current in the circuit so the equivalent inductance, L_T becomes equal to $(L \pm M) \div 2$.

Example No2

Two inductors whose self-inductances are of 75mH and 55mH respectively are connected together in parallel aiding. Their mutual inductance is given as 22.5mH. Calculate the total inductance of the parallel combination.

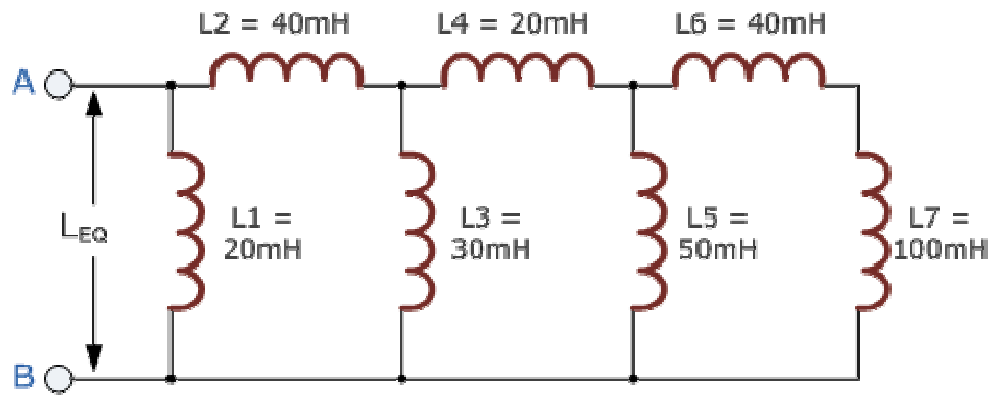
$$L_T = \frac{L_1 \times L_2 - M^2}{L_1 + L_2 - 2M}$$

$$L_T = \frac{75\text{mH} \times 55\text{mH} - 22.5\text{mH}^2}{75\text{mH} + 55\text{mH} - 2 \times 22.5\text{mH}}$$

$$L_T = 42.6\text{mH}$$

Example No3

Calculate the equivalent inductance of the following inductive circuit.



Calculate the first inductor branch, L_A

$$L_A = \frac{L5 \times (L6 + L7)}{L5 + L6 + L7} = \frac{50\text{mH} \times (40\text{mH} + 100\text{mH})}{50\text{mH} + 40\text{mH} + 100\text{mH}} = 36.8\text{mH}$$

Calculate the second inductor branch, L_B

$$L_B = \frac{L3 \times (L4 + L_A)}{L3 + L4 + L_A} = \frac{30\text{mH} \times (20\text{mH} + 36.8\text{mH})}{30\text{mH} + 20\text{mH} + 36.8\text{mH}} = 19.6\text{mH}$$

Calculate the equivalent circuit inductance, L_{EQ}

$$L_{EQ} = \frac{L1 \times (L2 + L_B)}{L1 + L2 + L_B} = \frac{20\text{mH} \times (40\text{mH} + 19.6\text{mH})}{20\text{mH} + 40\text{mH} + 19.6\text{mH}} = 15\text{mH}$$

Then the equivalent inductance is 15mH.

Inductors in Parallel Summary

As with the resistor, inductors connected together in parallel have the same voltage, V across them. Also connecting together inductors in parallel decreases the effective inductance of the circuit with the equivalent inductance of "N" inductors connected in parallel being the reciprocal of the sum of the reciprocals of the individual inductances. As with series connected inductors, mutually connected inductors in parallel are classed as either "aiding" or "opposing" this total inductance depending whether the coils are cumulatively coupled (in the same direction) or differentially coupled (in opposite direction).

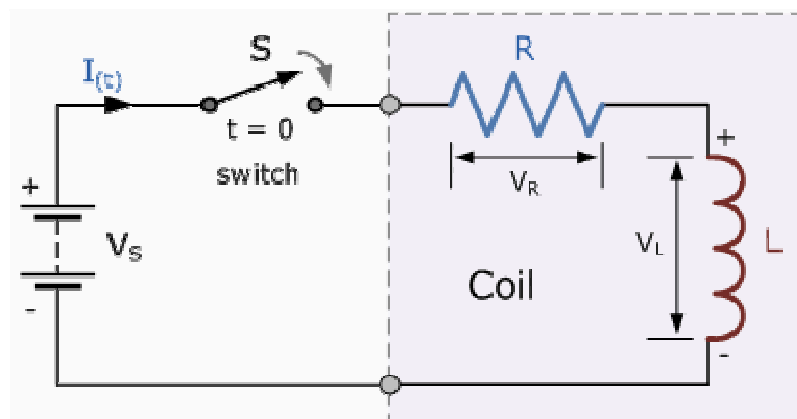
Thus far we have examined the inductor as a pure or ideal passive component. In the next tutorial about Inductors, we will look at non-ideal inductors that have real world resistive coils producing the equivalent circuit of an inductor in series with a resistance and examine the time constant of such a circuit.

The LR Series Circuit

In the first tutorial about [Inductors](#), we looked briefly at the time constant of an inductor stating that the current flowing through an inductor could not change instantaneously, but would increase at a constant rate determined by the self-induced emf in the inductor. In other words, an inductor in a circuit opposes the flow of current, (i) through it. While this is perfectly correct, we made the assumption in the tutorial that it was an ideal inductor which had no resistance or capacitance associated with its windings. However, in the real world "ALL" coils whether they are chokes, solenoids, relays or any wound component will always have a certain amount of resistance no matter how small associated with the coils turns of wire being used to make it as the copper wire will have a resistive value. Then we can consider our simple coil as being an "Inductance" in series with a "Resistance", in other words an **LR Series Circuit**.

An **LR Series Circuit** consists basically of an inductor of inductance L connected in series with a resistor of resistance R . The resistance R is the DC resistive value of the wire turns or loops that goes into making up the inductors coil. Consider the LR series circuit below.

The LR Series Circuit



The above *LR series circuit* is connected across a constant voltage source, (the battery) and a switch. Assume that the switch, S is open until it is closed at a time $t = 0$, and then remains permanently closed producing a "step response" type voltage input. The current, i begins to flow through the circuit but does not rise rapidly to its maximum value of I_{max} as determined by the ratio of V / R (Ohms Law). This limiting factor is due to the presence of the self induced emf within the inductor as a result of the growth of magnetic flux, (Lenz's Law). After a time the voltage source neutralizes the effect of the self induced emf, the current flow becomes constant and the induced current and field are reduced to zero.

We can use [Kirchoffs Voltage Law](#), (KVL) to define the individual voltage drops that exist around the circuit and then hopefully use it to give us an expression for the flow of current.

Kirchoffs voltage law gives us:

$$V_{(t)} = V_R + V_L = 0$$

The voltage drop across the resistor, R is IR (Ohms Law).

$$V_R = I \times R$$

The voltage drop across the inductor, L is by now our familiar expression $L = di/dt$

$$V_L = L \frac{di}{dt}$$

Then the final expression for the individual voltage drops around the LR series circuit can be given as:

$$V_{(t)} = I \times R + L \frac{di}{dt}$$

We can see that the voltage drop across the resistor depends upon the current, i, while the voltage drop across the inductor depends upon the rate of change of the current, di/dt. When the current is equal to zero, (i = 0) at time t = 0 the above expression, which is also a first order differential equation, can be rewritten to give the value of the current at any instant of time as:

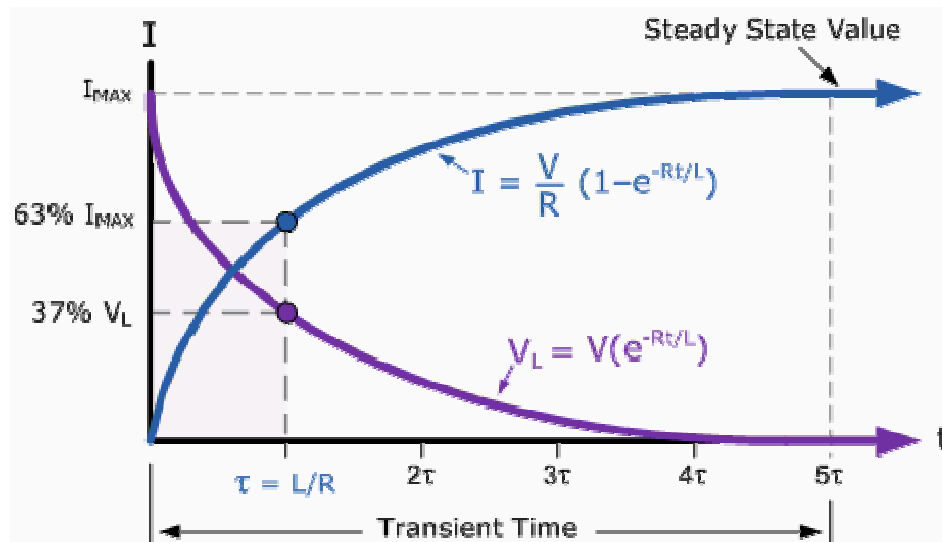
Expression for the Current in an LR Series Circuit

$$I_{(t)} = \frac{V}{R} \left(1 - e^{-Rt/L} \right) \text{ (A)}$$

- Where: V is in volts, R is in ohms, L is in henries, t is in seconds and e is the base of the Natural Logarithm = 2.71828

The L/R term in the above equation is known commonly as the **Time Constant**, (τ) of the LR series circuit and V/R also represents the final steady state current value in the circuit. Once the current reaches this maximum steady state value at 5τ , the inductance of the coil has reduced to zero acting more like a short circuit and effectively removing it from the circuit. Therefore the current flowing through the coil is limited only by the resistive element in Ohms of the coils windings. A graphical representation of the current growth representing the voltage/time characteristics of the circuit can be presented as.

Transient Curves for an LR Series Circuit



Since the voltage drop across the resistor, V_R is equal to $I \times R$ (Ohms Law), it will have the same exponential growth and shape as the current. However, the voltage drop across the inductor, V_L will have a value equal to: $V e^{(-Rt/L)}$. Then the voltage across the inductor, V_L will have an initial value equal to the battery voltage at time $t = 0$ or when the switch is first closed and then decays exponentially to zero as represented in the above curves.

The time required for the current flowing in the LR series circuit to reach its maximum steady state value is equivalent to about **5 time constants** or 5τ . This time constant τ , is measured by $\tau = L/R$, in seconds, where R is the value of the resistor in ohms and L is the value of the inductor in Henries. This then forms the basis of an RL charging circuit where 5τ can also be thought of as "5 x L/R " or the *transient time* of the circuit.

The transient time of any inductive circuit is determined by the relationship between the inductance and the resistance. For example, for a fixed value resistance the larger the inductance the slower will be the transient time and therefore a longer time constant for the LR series circuit. Likewise, for a fixed value inductance the smaller the resistance value the longer the transient time. However, for a fixed value inductance, by increasing the resistance value the transient time and therefore the time constant of the circuit becomes shorter. This is because as the resistance increases the circuit becomes more and more resistive as the value of the inductance becomes negligible compared to the resistance. If the value of the resistance is increased sufficiently large compared to the inductance the transient time would effectively be reduced to almost zero.

Example No1

A coil which has an inductance of 40mH and a resistance of 2Ω s is connected together to form a RL series circuit. If they are connected to a 20V DC supply.

a) What will be the final steady state value of the current.

Steady State Current, $I = \frac{V}{R} = \frac{20}{2} = 10A$

b) What will be the time constant of the RL series circuit.

Time Constant, $\tau = \frac{L}{R} = \frac{0.04}{2} = 0.02s \text{ or } 20ms$

c) What will be the transient time of the RL series circuit.

Transient Time, $5\tau = 5 \times 0.02s = 100ms$

c) What will be the value of the induced emf after 10mS.

Induced emf, $V_L = V e^{(-Rt/L)} = 20 e^{(-2 \times 0.01/0.04)}$

$$V_L = 20 \times 0.6065 = 12.13V$$

d) What will be the value of the circuit current one time constant after the switch is closed.

Instantaneous Current, $I_{(t)} = \frac{V_s}{R} (1 - e^{-Rt/L})$

The Time Constant, τ of the circuit was calculated in question b) as being 20mS. Then the circuit current at this time is given as:

$$I_{(t)} = \frac{20}{2} (1 - e^{-2 \times 0.02/0.04})$$

$$I_{(t)} = 10(1 - 0.368) = 6.32A$$

You may notice that the answer for question (d) which gives a value of 6.32 Amps at one time constant, is equal to 63.2% of the final steady state current value of 10 Amps we calculated in question (a). This value of 63.2% or $0.632 \times I_{MAX}$ also corresponds with the transient curves shown above.

Power in a LR Series Circuit

Then from above, the power in a LR series circuit is given as:

The instantaneous rate at which the voltage source delivers power to the circuit is given as:

$$P = V \times I \quad \text{in Watts}$$

The instantaneous rate at which power is dissipated by the resistor in the form of heat is given as:

$$P = I^2 \times R \quad \text{in Watts}$$

The rate at which energy is stored in the inductor in the form of magnetic potential energy is given as:

$$P = Vi = Li \frac{di}{dt} \quad \text{in Watts}$$

Then we can find the total power in a RL series circuit by multiplying by i and is therefore:

$$P = i^2 R + Li \frac{di}{dt} \quad (\text{Watts})$$

where the first term represents the power dissipated by the resistor (heat) and the second term represents the power absorbed by the inductor (magnetic energy).

Inductive Reactance

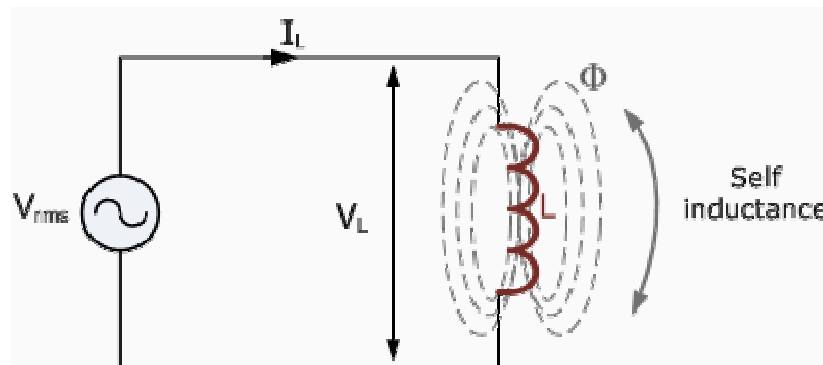
So far we have looked at inductors connected to DC networks and hopefully by now we know that when a DC voltage is applied across an inductor, the growth of the current through it is not instant but is determined by the inductors self-induced or back emf value. Also that the current continues to rise until it reaches its maximum steady state condition after five time constants. The maximum current flowing through the coil is limited only by the resistive element of the coils windings in Ohms, and which in turn is determined by the ratio of voltage over current, V/R , (Ohms Law).

When an alternating or AC voltage is applied across an inductor the flow of current through it behaves very differently to that of an applied DC voltage, by producing a phase difference between the voltage and the current waveforms. Now in an AC circuit, the opposition to current flow through the coils windings not only depends upon the inductance of the coil but also the frequency of the AC waveform. The opposition to current flowing through a coil in an AC circuit is determined by the AC resistance of the circuit. But resistance is always associated with DC circuits so to distinguish DC resistance from AC resistance which is also known as Impedance, the term **Reactance** is used. Like resistance, reactance is also measured in Ohm's but is given the symbol "X" to distinguish it from a purely resistive value and as the component is an inductor, the reactance of an inductor is called inductive reactance.

In other words, an inductor's resistance in an AC circuit is called **Inductive Reactance**.

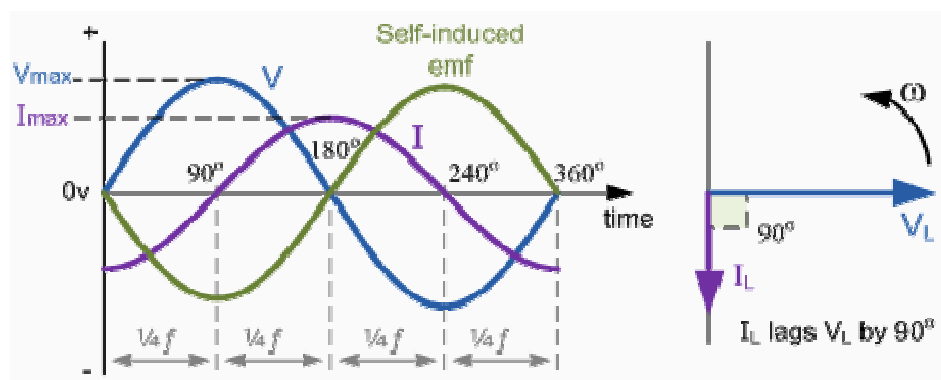
Inductive Reactance which is given the symbol X_L , is the property in an AC circuit which opposes the change in the current. In our tutorials about [Capacitors in AC Circuits](#), we saw that in a purely capacitive circuit, the current I_C "LEADS" the voltage by 90° . In a purely inductive AC circuit the exact opposite is true, the current I_L "LAGS" the applied voltage by 90° , or $(\pi/2 \text{ rads})$.

AC Inductor Circuit



In the purely inductive circuit above, the inductor is connected directly across the AC supply voltage. As the supply voltage increases and decreases with the frequency, the self-induced back emf also increases and decreases in the coil with respect to this change. We know that this self-induced emf is directly proportional to the rate of change of the current through the coil and is at its greatest as the supply voltage crosses over from its positive half cycle to its negative half cycle or vice versa at points, 0° and 180° along the sine wave. Consequently, the least voltage change occurs when the AC sine wave crosses over at its maximum or minimum peak voltage level. At these positions in the cycle the maximum or minimum currents are flowing through the inductor circuit and this is shown below.

AC Inductor Phasor Diagram



These voltage and current waveforms show that for a purely inductive circuit the current lags the voltage by 90° . Likewise, we can also say that the voltage leads the current by 90° . Either way the general expression is that the current lags as shown in

the vector diagram. Here the current vector and the voltage vector are shown displaced by 90° . *The current lags the voltage.*

We can also write this statement as, $V_L = 0^\circ$ and $I_L = -90^\circ$ with respect to the voltage, V_L . If the voltage waveform is classed as a sinewave then the current, I_L can be classed as a negative cosine and we can define the value of the current at any point in time as being:

$$I_L = I_{\max} \sin(\omega t - 90^\circ)$$

- Where: ω is in radians per second and t is in seconds.

Since the current always lags the voltage by 90° in a purely inductive circuit, we can find the phase of the current by knowing the phase of the voltage or vice versa. So if we know V_L , I_L must lag by 90° and if I_L is known then V_L must lead by 90° . Then this ratio of voltage to current in an inductive circuit will produce an equation that defines the **Inductive Reactance**, X_L of the coil.

Inductive Reactance

$$X_L = \frac{V_L}{I_L} = \omega L \ (\Omega)$$

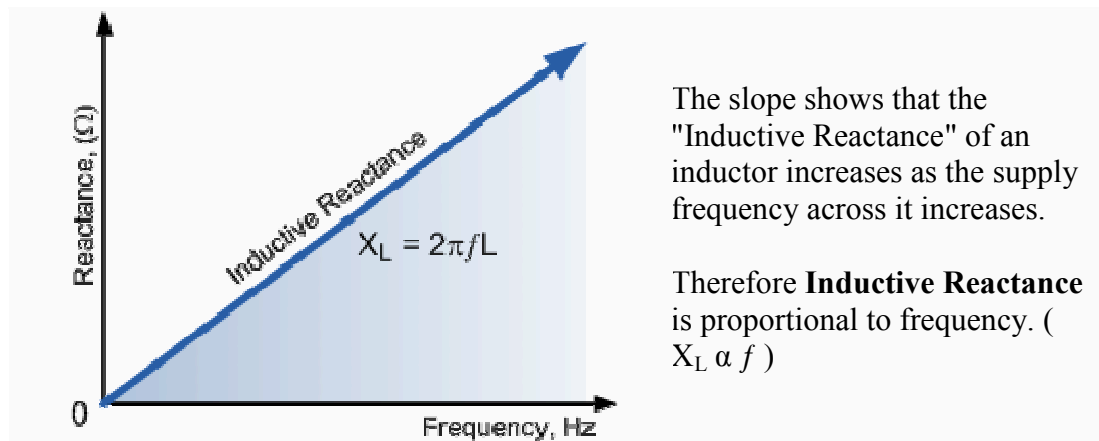
We can rewrite the above equation for inductive reactance into a more familiar form that uses the ordinary frequency of the supply instead of the angular frequency in radians, ω and this is given as:

$$X_L = 2\pi f L$$

- Where: f is the Frequency and L is the Inductance of the Coil and $2\pi f = \omega$.

From the above equation for inductive reactance, it can be seen that if either of the **Frequency** or **Inductance** was increased the overall inductive reactance value would also increase. As the frequency approaches infinity the inductors reactance would also increase to infinity acting like an open circuit. However, as the frequency approaches zero or DC, the inductors reactance would decrease to zero, acting like a short circuit. This means then that inductive reactance is "**Proportional**" to frequency and is small at low frequencies and high at higher frequencies and this demonstrated in the following graph:

Inductive Reactance against Frequency



Example No1

A coil of inductance 150mH and zero resistance is connected across a 100V, 50Hz supply. Calculate the inductive reactance of the coil and the current flowing through it.

Inductive Reactance:

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.12\Omega$$

Current:

$$I = \frac{V}{X_L} = \frac{100}{47.12} = 2.12A$$

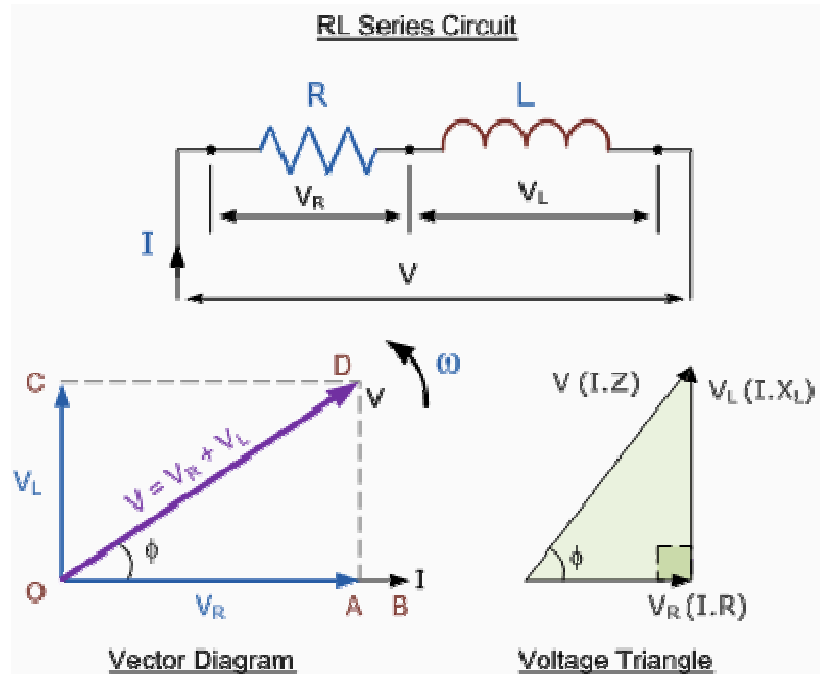
AC Through a RL Series Circuit

So far we have considered a purely inductive coil, but it is impossible to have a pure inductance as all coils, relays or solenoids will have a certain amount of resistance no matter how small associated with the coils turns of wire being used. Then we can consider our simple coil as being a resistance in series with an inductance. In an AC circuit that contains both inductance, L and resistance, R the voltage, V will be the phasor sum of the two component voltages, V_R and V_L . This means then that the current flowing through the coil will still lag the voltage, but by an amount less than 90° depending upon the values of V_R and V_L . The new angle between the voltage and the current is known as the phase angle of the circuit and is given the Greek symbol phi, Φ .

To be able to produce a vector diagram of the relationship between the voltage and the current, a reference or common component must be found. In a series connected R-L circuit the current is common as the same current flows through each component. The vector of this reference quantity is generally drawn horizontally from left to right. From our tutorials about resistors and capacitors we know that the current and voltage in a resistive AC circuit are both "in-phase" and therefore vector, V_R is drawn superimposed to scale on the current or reference line.

We also know from above, that the current "lags" the voltage in a purely inductive circuit and therefore vector, V_L is drawn 90° in front of the current reference and to the same scale as V_R and this is shown below.

RL Series AC Circuit



In the vector diagram above it can be seen that line OB represents the current reference line, line OA is the voltage of the resistive component and which is in-phase with the current. Line OC shows the inductive voltage which is 90° in front of the current, therefore it can be seen that the current lags the voltage by 90° . Line OD gives us the resultant or supply voltage across the circuit. The voltage triangle is derived from Pythagoras's theorem and is given as:

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$\text{and } \tan \Phi = \frac{V_L}{V_R}$$

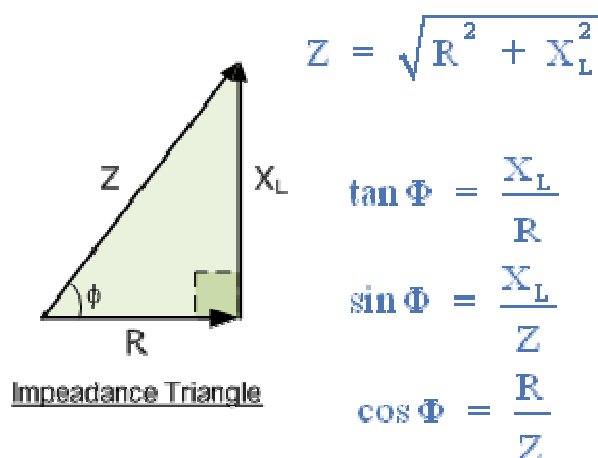
$$\text{Since } V_R = I \times R, \text{ and } V_L = I \times X_L$$

$$V = \sqrt{(I \times R)^2 + (I \times X_L)^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z} \quad (\text{A})$$

In a DC circuit, the ratio of voltage to current is called resistance. However, in an AC circuit this ratio is known as **Impedance**, **Z** with units again in Ohms. Impedance is the total resistance to current flow in an "AC circuit" containing both resistance and inductive reactance. If we divide the sides of the voltage triangle above by the current, another triangle is obtained whose sides represent the resistance, reactance and impedance of the coil. This new triangle is called an "Impedance Triangle"

The Impedance Triangle



Example No2

A solenoid coil has a resistance of 30 Ohms and an inductance of 0.5H. If the current flowing through the coil is 4 amps. Calculate,

a) The voltage of the supply if the frequency is 50Hz.

$$X_L = 2\pi fL$$

$$X_L = 2\pi \cdot 50 \cdot 0.5$$

$$X_L = 157\Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{30^2 + 157^2}$$

$$Z = 160\Omega$$

$$V = I \times Z = 4 \times 160 = 640V$$

b) The phase angle between the voltage and the current.

$$\tan \Phi = \frac{X_L}{R}$$

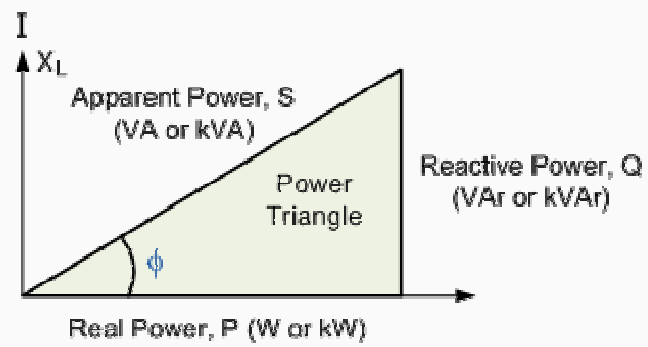
$$\therefore \Phi = \tan^{-1} \frac{X_L}{R} = \frac{157}{30} = 79.2^\circ \text{ lagging}$$

Power Triangle of an AC Inductor

There is one other type of triangle configuration that we can use for an inductive circuit and that is of the "Power Triangle". The power in an inductive circuit is known as **Reactive Power** or **volt-amps reactive**, symbol **Var** which is measured in volt-amps. In a RL series AC circuit, the current lags the supply voltage by an angle of Φ° . In a purely inductive AC circuit the power will be out of phase by 90° , because of the current but the total power of the coil will be equal to zero as any consumed power is cancelled out the generated self-induced emf power. In other words, energy is both taken from the supply and returned to it.

The Reactive Power, (Q) of a coil can be given as: $I^2 \times X_L$ (similar to $I^2 R$ in a DC circuit). Then the three sides of a power triangle in an AC circuit are represented by apparent power, (S), real power, (P) and the reactive power, (Q) as shown.

Power Triangle



$$P = VI \cos \phi, \quad S = VI, \quad Q = VI \sin \phi$$

$$VA = \sqrt{P^2 + VAR^2}$$